Properties of the Effective Hamiltonian for the System of Neutral Kaons*

Justyna Jankiewicz
Instytute of Physics
University of Zielona Gora
Prof. Z. Szafrana 4a, Zielona Gora, Poland
J.Jankiewicz@proton.if.uz.zgora.pl

09-05-2004 - 12-04-2004

Abstract

We study the properties of time evolution of the $K^0 - \bar{K}^0$ system in spectral formulation. Within the one–pole model we find the exact form of the diagonal matrix elements of the effective Hamiltonian for this system. It appears that, contrary to the Lee–Oehme–Yang (LOY) result, these exact diagonal matrix elements are different if the total system is CPT–invariant but CP–noninvariant.

1 Introduction

Following the LOY approach, a nonhermitian Hamiltonian $H_{\|}$ is usually used to study the properties of the particle-antiparticle unstable system [2] - [6]

$$H_{\|} \equiv M - \frac{i}{2}\Gamma, \tag{1}$$

where

$$M = M^{+}\ ,\ \Gamma = \Gamma^{+} \tag{2}$$

are (2×2) matrices acting in a two-dimensional subspace $\mathcal{H}_{\|}$ of the total state space $\mathcal{H}$. The M-matrix is called the mass matrix and Γ is the decay matrix. Lee, Oehme and Yang derived their approximate effective Hamiltonian $H_{\|} \equiv H_{LOY}$ by adapting the one-dimensional Weisskopf-Wigner (WW) method to the two-dimensional case corresponding to the neutral kaon system.
Almost all properties of this system can be described by solving the Schrödinger-like equation [2] - [5]

$$i\frac{\partial}{\partial t}|\psi;t\rangle_{\|} = H_{\|}|\psi;t\rangle_{\|}, \quad (t \geq t_0 > -\infty) \tag{3}$$

*Paper presented at **The XXXVI Symposium on Mathematical Physics**, *Poster session*, Toruń, Poland, June 9-12, 2004 . This is shortened version of [1]

(where we have used $\hbar = c = 1$) with the initial conditions

$$\parallel |\psi; t = t_0\rangle_{\parallel} \parallel = 1, \quad |\psi; t_0 = 0\rangle_{\parallel} = 0, \tag{4}$$

for $|\psi; t = t_0\rangle_{\parallel}$ belonging to the subspace of states $\mathcal{H}_{\parallel}$ ($\mathcal{H}_{\parallel} \subset \mathcal{H}$), spanned by, e.g., orthonormal neutral kaons states K^0 and $\bar{K}^0$. The solutions of Eq. (3) may be written in a matrix form, which may be used to define the time evolution operator $U_{\parallel}(t)$ acting in subspace $\mathcal{H}_{\parallel}$

$$|\psi; t\rangle_{\parallel} = U_{\parallel}(t)|\psi; t_0 = 0\rangle_{\parallel} \equiv U_{\parallel}(t)|\psi\rangle_{\parallel}, \tag{5}$$

where

$$|\psi\rangle_{\parallel} \equiv a_1|\mathbf{1}\rangle + a_2|\mathbf{2}\rangle \tag{6}$$

and $|\mathbf{1}\rangle$ denotes particle "1" – in the present case $|K^0\rangle$ whereas $|\mathbf{2}\rangle$ corresponds to the antiparticle state for particle "1": $|\bar{K}^0\rangle$, $\langle \mathbf{j}|\mathbf{k}\rangle = \delta_{jk}$, $j, k = 1, 2$. It is usually assumed that the real parts of the diagonal matrix elements of $H_{\parallel}$, namely $\Re(\cdot)$,

$$\Re(h_{jj}) \equiv M_{jj} \quad (j = 1, 2), \tag{7}$$

where

$$h_{jk} = \langle \mathbf{j}|H_{\parallel}|\mathbf{k}\rangle \qquad (j, k = 1, 2) \tag{8}$$

correspond to the masses of the particle "1" and its antiparticle "2" [2] - [6]. $\Im(\cdot)$ is the imaginary part of h_{jj}

$$\Im(h_{jj}) \equiv \Gamma_{jj} \quad (j = 1, 2) \tag{9}$$

and Γ_{jj} are interpreted as the decay widths of the particles. According to the standard result of the LOY approach, in a CPT invariant system, i.e. when

$$\Theta H \Theta^{-1} = H, \tag{10}$$

(where $\Theta = CPT$, $H = H^+$ is the Hamiltonian of the total system under consideration)
we have

$$h_{11}^{LOY} = h_{22}^{LOY}. \tag{11}$$

The universal properties of the unstable particle-antiparticle subsystem described by the H fulfilling the condition (10), may be investigated by using the matrix elements of the exact $U_{\parallel}$, instead of the approximate one used in the LOY theory. The exact $U_{\parallel}$ can be written as follows

$$U_{\parallel}(t) = PU(t)P, \tag{12}$$

where

$$P \equiv |\mathbf{1}\rangle\langle\mathbf{1}| + |\mathbf{2}\rangle\langle\mathbf{2}|, \tag{13}$$

and $U(t)$ is the exact evolution operator acting in the whole state space. This operator is the solution of the Schrödinger equation

$$i\frac{\partial}{\partial t}U(t)|\phi\rangle = HU(t)|\phi\rangle, \quad U(0) = I. \tag{14}$$

I is the unit operator in the $\mathcal{H}$ space and $|\phi\rangle \equiv |\phi; t_0 = 0\rangle \in \mathcal{H}$ is the initial state of the system.
In the remaining part of the poster we will be using the following matrix representation of the evolution operator

$$U_{\|}(t) \equiv \left(\begin{array}{cc} \mathbf{A(t)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right), \tag{15}$$

where $\mathbf{0}$ denotes the zero submatrices of the suitable dimension, and the $\mathbf{A(t)}$ is a (2×2) matrix acting in $\mathcal{H}_{\|}$

$$\mathbf{A(t)} = \left(\begin{array}{cc} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{array} \right), \tag{16}$$

where

$$A_{jk}(t) = \langle \mathbf{j}|U_{\|}(t)|\mathbf{k}\rangle \equiv \langle \mathbf{j}|U(t)|\mathbf{k}\rangle \quad (j,k = 1,2). \tag{17}$$

Assuming that the property (10) holds and using the following definitions

$$\Theta|\mathbf{1}\rangle \equiv e^{-i\theta}|\mathbf{2}\rangle, \quad \Theta|\mathbf{2}\rangle \equiv e^{-i\theta}|\mathbf{1}\rangle, \tag{18}$$

it can be shown that

$$A_{11}(t) = A_{22}(t). \tag{19}$$

A very important relation between the amplitudes $A_{12}(t)$ and $A_{21}(t)$ follows from the famous Khalfin Theorem [7] - [9]

$$r(t) \equiv \frac{A_{12}(t)}{A_{21}(t)} = const \equiv r \quad \Rightarrow \quad |r| = 1. \tag{20}$$

General conclusions concerning the properties of the matrix elements of $H_{\|}$ can be drawn by analyzing the following identity [4, 11]

$$H_{\|}(t) \equiv i \frac{\partial \mathbf{A}(t)}{\partial t} [\mathbf{A}(t)]^{-1}. \tag{21}$$

Using Eq. (21) we can easily find the general formulae for the diagonal matrix elements h_{jj}, of $H_{\|}(t)$ and next assuming (10) and using relation (19) which follows from our earlier assumptions, we get

$$h_{11}(t) - h_{22}(t) = \frac{i}{det\mathbf{A}(t)} \left(\frac{\partial A_{21}(t)}{\partial t} A_{12}(t) - \frac{\partial A_{12}(t)}{\partial t} A_{21}(t) \right). \tag{22}$$

In [11] it was shown, by using relation (22), that this result means that in the considered case (with CPT conserved) for $t > 0$ we get the following theorem

$$h_{11}(t) - h_{22}(t) = 0 \quad \Leftrightarrow \quad \frac{A_{12}(t)}{A_{21}(t)} = const \quad (t > 0). \tag{23}$$

Thus, for $t > 0$ the problem under study is reduced to the Khalfin Theorem (see relation (20)) [11].

Having noticed this, let us now turn our attention to the conclusions following from Khalfin's Theorem. CP noninvariance requires that $|r| \neq 1$ [2, 3, 4, 5, 7, 9, 12, 13]. This means that in this case the following condition must be fulfilled: $r = r(t) \neq const$. Consequently, if in the considered system property (10) holds, but at the same time

$$[\mathcal{CP}, H] \neq 0 \tag{24}$$

and the unstable states "1" i "2" are connected by (18), then in this system for $t > 0$ [11]

$$h_{11}(t) - h_{22}(t) \neq 0. \tag{25}$$

So, in the exact quantum theory the difference $(h_{11}(t) - h_{22}(t))$ cannot be equal to zero with CPT conserved and CP violated.

2 A model: one pole approximation

While describing the two and three pion decay we are mostly interested in the $|K_S\rangle$ and $|K_L\rangle$ superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$. These states correspond to the physical $|K_S\rangle$ and $|K_L\rangle$ neutral kaon states [13, 14]

$$|K_S\rangle = p|K^0\rangle + q|\bar{K}^0\rangle, \quad |K_L\rangle = p|K^0\rangle - q|\bar{K}^0\rangle. \tag{26}$$

Using the spectral formalism we can write an unstable state $|\lambda(t)\rangle$ as

$$|\lambda(t)\rangle = \sum_q |q(t)\rangle \omega_\lambda(q), \tag{27}$$

where $|q(t)\rangle = e^{-itH}|q\rangle$, vectors $|q\rangle$ form a complete set of eigenvectors of the hermitian, quantum-mechanical Hamiltonian H and $\omega_\lambda(q) = \langle q|\lambda\rangle$. If the continuous eigenvalue is denoted by m, we can define the survival amplitude $A(t)$ (or the transition amplitude in the case of $K^0 \leftrightarrow \bar{K}^0$) in the following way:

$$A(t) = \int\limits_{Spec(H)} dm\; e^{-imt}\rho(m), \tag{28}$$

where the integral extends over the whole spectrum of the Hamiltonian and density $\rho(m)$ is defined as follows

$$\rho(m) = |\omega_\lambda(m)|^2, \tag{29}$$

where $\omega_\lambda(m) = \langle m|\lambda\rangle$.

In accordance with formula (27) the unstable states K_S and K_L may now be written as a superposition of the eigenkets

$$|K_S\rangle = \int_0^\infty dm\; \sum_\alpha \omega_{S,\alpha}(m)|\phi_\alpha(m)\rangle; \tag{30}$$

$$|K_L\rangle = \int_0^\infty dm\; \sum_\beta \omega_{L,\beta}(m)|\phi_\beta(m)\rangle. \tag{31}$$

The Breit-Wigner ansatz [15]

$$\rho_{WB}(m) = \frac{\Gamma}{2\pi} \frac{1}{(m-m_0)^2 + \frac{\Gamma^2}{4}} \equiv |\omega(m)|^2 \tag{32}$$

leads to the well known exponential decay law which follows from the survival amplitude

$$A_{BW}(t) = \int_{-\infty}^{\infty} dm\, e^{-imt} \rho_{WB}(m) = e^{-im_0 t} e^{-\frac{1}{2}\Gamma|t|}. \tag{33}$$

(Note that the existence of the ground state induces non-exponential corrections to the decay law and to the survival amplitude (33) — see [13]). It is therefore reasonable to assume a suitable form for $\omega_{S,\beta}$ and $\omega_{L,\beta}$. More specifically, we write [13]

$$\omega_{S,\beta}(m) = \sqrt{\frac{\Gamma_S}{2\pi}} \frac{A_{S,\beta}(K_S \to \beta)}{m - m_S + i\frac{\Gamma_S}{2}}, \quad \omega_{L,\beta}(m) = \sqrt{\frac{\Gamma_L}{2\pi}} \frac{A_{L,\beta}(K_L \to \beta)}{m - m_L + i\frac{\Gamma_L}{2}} \tag{34}$$

where $A_{S,\beta}$ and $A_{L,\beta}$ are decay (transition) amplitudes, end thus

$$\rho_{x,\beta}(m) = \frac{\Gamma_x}{2\pi} \frac{(A_{x,\beta}(K_x \to \beta))^2}{(m - m_x)^2 + \frac{(\Gamma_x)^2}{4}}, \tag{35}$$

where $x = L, S$.

In the one-pole approximation (34) $A_{K^0K^0}(t)$ can be conveniently written as

$$\begin{aligned} A_{K^0K^0}(t) &= A_{\bar{K}^0\bar{K}^0}(t) = \\ &= -\frac{1}{2\pi}\Bigg\{ e^{-im_S t} \left(-\int_0^{-\frac{m_S}{\gamma_S}} dy \frac{e^{-i\gamma_S t y}}{y^2+1} + \int_0^{\infty} dy \frac{e^{-i\gamma_S t y}}{y^2+1} \right) + \\ &\quad + e^{-im_L t} \left(-\int_0^{-\frac{m_L}{\gamma_L}} dy \frac{e^{-i\gamma_L t y}}{y^2+1} + \int_0^{\infty} dy \frac{e^{-i\gamma_L t y}}{y^2+1} \right) \Bigg\}. \end{aligned} \tag{36}$$

Collecting only exponential terms in (36) one obtains an expression analogous to the WW approximation

$$A_{K^0K^0}(t) = A_{\bar{K}^0\bar{K}^0}(t) = \frac{1}{2} \left(e^{-im_S t} e^{-\gamma_S t} + e^{-im_L t} e^{-\gamma_L t} \right) + N_{K^0K^0}(t). \tag{37}$$

Here $N_{K^0K^0}(t)$ denotes all non-oscillatory terms present in the integral (36).

3 Diagonal matrix elements of the effective Hamiltonian

Using the decomposition of type (37) and the one-pole ansatz (34), we find the difference (25), which is now formulated for the $K^0 - \bar{K}^0$ system. Here it has

the following form:

$$h_{11}(t) - h_{22}(t) = \frac{X(t)}{Y(t)}, \tag{38}$$

where

$$X(t) = i \left(\frac{\partial A_{\bar{K}^0 K^0}(t)}{\partial t} A_{K^0 \bar{K}^0}(t) - \frac{\partial A_{K^0 \bar{K}^0}(t)}{\partial t} A_{\bar{K}^0 K^0}(t) \right) \tag{39}$$

and

$$Y(t) = A_{K^0 K^0}(t) A_{\bar{K}^0 \bar{K}^0}(t) - A_{K^0 \bar{K}^0}(t) A_{\bar{K}^0 K^0}(t). \tag{40}$$

Using the above mentioned spectral formulae in the one - pole approximation (34) we get $A_{K^0 \bar{K}^0}(t)$ and $A_{\bar{K}^0 K^0}(t)$

$$\begin{aligned} A_{K^0 \bar{K}^0}(t) = \frac{1+\pi}{8\pi p^* q} \Bigg\{ & e^{-i m_S t} e^{-\gamma_S t} \Bigg[1 + \\ & + \frac{\sqrt{\gamma_S \gamma_L}}{\gamma_S} \left(-2\, i\, \gamma_S C_I + D'_I - F'_I \right) \Bigg] + \\ & + e^{-i m_L t} e^{-\gamma_L t} \Bigg[-1 + \\ & + \frac{\sqrt{\gamma_S \gamma_L}}{\gamma_L} \left(2\, i\, \gamma_L C_I - D'_I + F'_I \right) \Bigg] \Bigg\} + \\ & + N_{K^0 \bar{K}^0}(t) \end{aligned} \tag{41}$$

and

$$\begin{aligned} A_{\bar{K}^0 K^0}(t) = \frac{1+\pi}{8\pi p q^*} \Bigg\{ & e^{-i m_S t} e^{-\gamma_S t} \Bigg[1 + \\ & + \frac{\sqrt{\gamma_S \gamma_L}}{\gamma_S} \left(2\, i\, \gamma_S C_I - D'_I + F'_I \right) \Bigg] + \\ & + e^{-i m_L t} e^{-\gamma_L t} \Bigg[-1 + \\ & + \frac{\sqrt{\gamma_S \gamma_L}}{\gamma_L} \left(-2\, i\, \gamma_L C_I + D'_I - F'_I \right) \Bigg] \Bigg\} + \\ & + N_{\bar{K}^0 K^0}(t), \end{aligned} \tag{42}$$

where $N_{K^0 \bar{K}^0}(t)$, $N_{\bar{K}^0 K^0}(t)$ denotes all non-oscillatory terms and C_I, D'_I, F'_I are defined in [13].

Using the expression for the derivative of E_i we can find the derivatives which will be necessary for the following calculations $\frac{\partial A_{K^0\bar{K}^0}(t)}{\partial t}$ and $\frac{\partial A_{\bar{K}^0K^0}(t)}{\partial t}$:

$$\begin{aligned}\frac{\partial A_{K^0\bar{K}^0}(t)}{\partial t} &= \frac{1+\pi}{8\pi p^* q}\Bigg\{e^{-im_S t}e^{-\gamma_S t}\Bigg[-im_S-\gamma_S+\\ &+\sqrt{\gamma_S\gamma_L}\Bigg(2i\gamma_S C_I - D_I' + F_I'\Bigg)\Bigg]+\\ &+e^{-im_L t}e^{-\gamma_L t}\Bigg[im_L-\gamma_L+\\ &+\sqrt{\gamma_S\gamma_L}\Bigg(-2i\gamma_L C_I + D_I' - F_I'\Bigg)\Bigg]\Bigg\}+\\ &+\Delta N_{K^0\bar{K}^0}(t) \qquad (43)\end{aligned}$$

and

$$\begin{aligned}\frac{\partial A_{\bar{K}^0K^0}(t)}{\partial t} &= \frac{1+\pi}{8\pi p q^*}\Bigg\{e^{-im_S t}e^{-\gamma_S t}\Bigg[-im_S-\gamma_S+\\ &+\sqrt{\gamma_S\gamma_L}\Bigg(-2i\gamma_S C_I + D_I' - F_I'\Bigg)\Bigg]+\\ &+e^{-im_L t}e^{-\gamma_L t}\Bigg[im_L-\gamma_L+\\ &+\sqrt{\gamma_S\gamma_L}\Bigg(2i\gamma_L C_I - D_I' + F_I'\Bigg)\Bigg]\Bigg\}+\\ &+\Delta N_{\bar{K}^0K^0}(t), \qquad (44)\end{aligned}$$

where $\Delta N_{K^0\bar{K}^0}(t)$, $\Delta N_{\bar{K}^0K^0}(t)$ denotes all non-oscillatory terms.

The states $|K_L\rangle$ and $|K_S\rangle$ are superpositions of $|K^0\rangle$ and $|\bar{K}^0\rangle$. The lifetimes of particles $|K_L\rangle$ and $|K_S\rangle$ may be denoted by τ_L and τ_S, respectively, $\tau_L = \frac{1}{\gamma_L} = 5,183\cdot 10^{-8}s$ being much longer than $\tau_S = \frac{1}{\gamma_S} = 0,8923\cdot 10^{-10}s$.

Below we calculate the difference (38) for $t \sim \tau_L$

$$h_{11}(t\sim\tau_L) - h_{22}(t\sim\tau_L) = \frac{X(t\sim\tau_L)}{Y(t\sim\tau_L)}. \qquad (45)$$

If we only consider the long living states $|K_L\rangle$ we may drop all the terms containing $e^{-\gamma_S t}|_{t\sim\tau_L}$ as they are negligible in comparison with elements involving the factor $e^{-\gamma_L t}|_{t\sim\tau_L}$. We also drop all the non-oscillatory terms $N_{K^0K^0}(t)$, $N_{\bar{K}^0K^0}(t)$, $N_{K^0\bar{K}^0}(t)$ present in $A_{K^0K^0}(t)$, $A_{\bar{K}^0K^0}(t)$ and $A_{K^0\bar{K}^0}(t)$, that is in integrals (36), (41) and (42), because they are extremally small in the region of time $t\sim\tau_L$ [13, 16, 17]. Similarly, because of the properties of the exponential integral function E_i, we can drop terms like $\Delta N_{\bar{K}^0K^0}$ and $\Delta N_{K^0\bar{K}^0}$ present

in $\frac{\partial A_{\bar{K}^0 K^0}}{\partial t}$ (43) and $\frac{\partial A_{K^0 \bar{K}^0}}{\partial t}$ (44).This conclusion follows from the asymptotic properties of the exponential integral function E_i and the fact that $\Delta N_{\bar{K}^0 K^0}$, $\Delta N_{K^0 \bar{K}^0}$ only contain expressions proportional to E_i.

We may now calculate the products $A_{K^0 K^0}(t) A_{\bar{K}^0 \bar{K}^0}(t)$, $A_{K^0 \bar{K}^0}(t) A_{\bar{K}^0 K^0}(t)$, $\frac{\partial A_{\bar{K}^0 K^0}}{\partial t}(t) A_{K^0 \bar{K}^0}(t)$, $\frac{\partial A_{K^0 \bar{K}^0}}{\partial t}(t) A_{\bar{K}^0 K^0}(t)$, which, after using the above mentioned properties of $N_{K^0 K^0}(t)$, $\Delta N_{K^0 K^0}(t)$ and performing some algebraic transformations, leads to the following form of the difference (45):

$$h_{11}(t \sim \tau_L) - h_{22}(t \sim \tau_L)) = \left(\frac{2\pi^2 \sqrt{\gamma_S \gamma_L}}{\pi^2 + 2\pi + 1} \right) \cdot \frac{Z}{W} \neq 0, \qquad (46)$$

where

$$\begin{aligned} Z = \quad & 4|p|^2|q|^2 - \frac{\pi^2 + 2\pi + 1}{4\pi^2} \Bigg[1 + \\ & + \gamma_S \Bigg(4\gamma_L C_I^2 + \frac{1}{\gamma_L} (-D_I^{'2} - F_I^{'2} + 4 D_I^{'} F_I^{'}) + \\ & + 4 i C_I (D_I^{'} - F_I^{'}) \Bigg) \Bigg] \neq 0 \end{aligned} \qquad (47)$$

$$\begin{aligned} W = \quad & 2 \left(-C_I m_L + D_I^{'} - F_I^{'} \right) + \\ & + i \left[-4 C_I \gamma_L + \frac{m_L}{\gamma_L} \left(-D_I^{'} + F_I^{'} \right) \right] \neq 0. \end{aligned} \qquad (48)$$

4 Final remarks

- Our results presented in the present poster have shown that in a CPT invariant and CP noninvariant system in the case of the exactly solvable one-pole model, the diagonal matrix elements do not have to be equal. In the general case the diagonal elements depend on time and their difference, for example at $t \sim \tau_L$, is different from zero. Z and W in (46) are different from zero, so the difference $(h_{11}(t) - h_{22}(t))|_{t \sim \tau_L} \neq 0$. From this observation a conclusion of major importance can be drawn, namely that the measurement of the mass difference $(m_{K^0} - m_{\bar{K}^0})$ should not be used while designing CPT invariance tests. This runs counter to the general conclusions following from the Lee, Oehme and Yang theory.

- A detailed analysis of $h_{jk}(t)$, $(j, k = 1, 2)$ shows that the non-oscillatory elements $N_{\alpha,\beta}(t), \Delta N_{\alpha,\beta}(t)$ (where $\alpha, \beta = K^0, \overline{K}^0$) is the source of the non-zero difference $(h_{11}(t) - h_{22}(t))$ in the model considered. It is not difficult to verify that dropping all the terms of $N_{\alpha,\beta}(t), \Delta N_{\alpha,\beta}(t)$ type

in the formula for $(h_{11}(t) - h_{22}(t))$ gives $(h_{11}^{osc}(t) - h_{22}^{osc}(t)) = 0$, where $h_{jj}^{osc}(t)$, $(j = 1, 2)$, stands for $h_{jj}(t)$ without the non-oscillatory terms.

- The result $(h_{11}(t) - h_{22}(t)) \neq 0$ seems to be very important as it has been obtained within the exactly solvable one-pole model based on the Breit-Wigner ansatz, i.e. the same model as used by Lee, Oehme and Yang.

Acknowledgements

The author wishes to thank Professor Krzysztof Urbanowski for many helpful discussions.

References

[1] J. Jankiewicz, Diagonal Matrix Elements of the Effective Hamiltonian for $K^0 - \bar{K}^0$ System in One Pole Approximation, (Preprint hep-ph/0402268 v1, February 2004).

[2] T.D. Lee, R. Oehme, C. N. Yang, Phys. Rev., **106** (1957) 340.

[3] T.D. Lee, C.S. Wu, Annu. Rev. Nucl. Sci., **16** (1966) 471;
M.K. Gaillard, M. Nicolic (Eds.), Weak Interaction, INPN et de Physique des Particules, Paris, 1977, Ch. 5, Appendix A;
S.M. Bilenkij, in: Particles and Nucleus, Vol. 1. (1), 1970, p. 227, [in Russian].

[4] L.P. Horwitz, J.P. Marchand, Helv. Phys. Acta, **42** (1969) 801.

[5] J.W. Cronin, Acta Phys.Polon., **B 15** (1984) 419;
V.V. Barmin *et al*, Nucl.Phys., **B 247** (1984) 428;
L. Lavoura, Ann.Phys. (N.Y.), **207** (1991) 428;
C. Buchanan *et al*, Phys.Rev., **D 45** (1992) 4088;
C.O. Dib,R.D. Peccei, Phys.Rev., **D 46** (1992) 2265;
M. Zrałek, Acta Phys. Polon., **B 29** (1998) 3925;
M.Nowakowski, Mod. Phys. Lett. **A 17** (2002) 2039.

[6] K.Urbanowski, J.Piskorski, Found.Phys., **30**(2000)839,physics/9803030.

[7] L.A. Khalfin, Preprints of the University of Texas at Austin: New Results on the CP-violation problem (Report DOE-ER40200-211, February 1990);
L.A. Khalfin, A new CP-violation effect and new possibility for investigation of $K_S^0, K_L^0 (K^0, \bar{K}^0)$ decay modes (Report DOE-ER40200-247, Frebruary 1991).

[8] P.K. Kabir, A. Pilaftsis, Phys. Rev., **A 53** (1996) 66.

[9] L.A. Khalfin, Found. Phys., **27** (1997) 1549, and references one can find therein.

[10] O. Nachtmann, Elementary Particle Physics, Springer Verlag, Berlin 1990.

[11] K. Urbanowski, Phys. Lett., **B 540** (2002) 89.

[12] K. Hagiwara *et al*, Review of Particle Physics, Physical Review **D 66**, Part 1, No 1–I, (2002), 010001.

[13] M.Nowakowski, Time Evolution of $K^0 - \bar{K}^0$ System in Spectral Formulation, SIS-Pubblicazioni, LNF-96/004(P); M.Nowakowski, Int. J. Mod. Phys., **A 14** (1999) 589 .

[14] P.K. Kabir, The CP Puzzle, Academic Press, London 1968.

[15] A. Bohm, Quantum Mechanics: Foundations and Applications, Springer Verlag, Berlin 1986.

[16] I.S. Gradshteyn, I.M. Ryzhik, Tables of Integrals, Series and Products, 4th edition, Academic Press, London 1965.

[17] M. Abramowitz and I.A. Stegun, Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables, National Bureau of Standards Applied Mathematics Series - 55, Issued June 1964 Tenth Printing, December 1972, with corrections.